Q: Evaluate (i) $\int_0^{\pi/2} \sin^4 x \cos^3 x \, dx$

(ii) $\int_0^{\pi/2} \sin^6 x \cos^8 x \, dx$

$$= -(n-1)\sin^{m+2} x \cos^{n-2} x + (m+1)\sin^m x \cos^n x$$

$$\sin^m x \quad \sin^2 x \rightarrow (1-\cos^2 x) = \sin^m x(1-\cos^2 x)$$

$$= -(n-1)\sin^m x \cos^{n-2} x + (n-1)\sin^m x \cos^n x + (m+1)\sin^m x \cos^n x$$

$$\frac{dP}{dx} = -(n-1)\sin^m x \cos^{n-2} x + (m+n)\sin^m x \cos^n x$$

Integrating both sides, we get.

$$P = -(n-1)\int \sin^m x \cos^{n-2} x \, dx + (m+n)\int \sin^m x \cos^n x \, dx$$

or

$$(m+n)\int \sin^m x \cos^n x \, dx = P + (n-1)\int \sin^m x \cos^{n-2} x \, dx$$

or

$$\int \sin^m x \cos^n x \, dx = \frac{\sin^{m+1} x \cos^{n-1} x}{m+n} + \frac{(n-1)}{(m+n)}\int \sin^m x \cos^{n-2} x \, dx$$

$$\int_0^{\pi/2} \sin^m x \cos^n x \, dx = \left. \cancel{\frac{\sin^{m+1} x \cos^{n-1} x}{m+n}} \right|_0^{\pi/2} + \frac{n-1}{(m+n)}\int_0^{\pi/2} \sin^m x \cos^{n-2} x \, dx$$

$$\int_0^{\pi/2} \sin^m x \cos^n x \, dx = \frac{n-1}{m+n} \int_0^{\pi/2} \sin^m x \cos^{n-2} x \, dx \quad -(1)$$

$$\Downarrow \qquad\qquad\qquad\qquad \Downarrow$$

$$I_{m,n} \qquad\qquad\qquad\qquad I_{m,n-2}$$

$$I_{m,n} = \frac{n-1}{m+n} I_{m,n-2} \quad -(2)$$

$n \longrightarrow n-2$

$$I_{m,n-2} = \frac{n-3}{m+n-2} I_{m,n-4} \quad \text{putting in (2)}$$

$n \longrightarrow n-2$

$$I_{m,n-4} = \frac{n-5}{m+n-4} I_{m,n-6}$$

$$I_{m,n} = \frac{(n-1)(n-3)}{(m+n)(m+n-2)} I_{m,n-4}$$

$$I_{m,n} = \frac{(n-1)(n-3)(n-5)}{(m+n)(m+n-2)(m+n-4)} I_{m,n-6}$$

$n = 6$

$m \to$ odd, $m \to$ even

**Case 1:** let $n$ be even.

$$I_{m,n} = \frac{(n-1)(n-3)\ldots 1}{(m+n)(m+n-2)\ldots(m+2)} I_{m,0} \quad -(3)$$

$$I_{m,0} = \int_0^{\pi/2} \sin^m x \, dx = \begin{cases} \frac{(m-1)(m-3)\ldots 1}{m(m-2)\ldots 2}\left(\frac{\pi}{2}\right) & m \text{ is even} \quad (4) \\ \frac{(m-1)(m-3)\ldots \times 2}{m(m-2)\ldots \times 3} & m \text{ is odd} \quad (5) \end{cases}$$

(i) If $m$ & $n$ are both even

$$\int_0^{\pi/2} \sin^m x \cos^n x \, dx = \frac{(n-1)(n-3)\cdots 1 \times (m-1)(m-3)\cdots 1}{(m+n)(m+n-2)\cdots(m-2)\,m\,(m-2)\cdots 2} \times \frac{\pi}{2} \quad -(6)$$

(ii) If $m$ is odd and $n$ is even

$$\int_0^{\pi/2} \sin^m x \cos^n x \, dx = \frac{(n-1)(n-3)\cdots 1 \times (m-1)\cdots 2}{(m+n)(m+n-2)\cdots \times (m+2)\,m \cdots 3} \quad -(7)$$

<u>Case 2</u>: let $n$ be odd $<$ $m$ odd / $m$ even