$m \quad n-2$

$$I_{m,n} = \int \underset{\mathrm{I}}{\cos^m x}\ \underset{\mathrm{II}}{\sin nx}\, dx$$

$$= \cos^m x \left(-\frac{\cos nx}{n}\right) - \int m \cos^{m-1} x \left(-\frac{\cos nx}{n}\right)(-\sin x)\,dx$$

$$= -\frac{\cos^m x \cos nx}{n} - \frac{m}{n} \int \cos^{m-1} x \,(\sin x \cdot \cos nx)\, dx \quad \text{— (1)}$$

$\sin(A-B)$

$$\sin(n-1)x = \sin nx \cos x - \cos nx \sin x$$

or $\cos nx \sin x = \sin nx \cos x - \sin(n-1)x$

putting in (1)

$$I_{m,n} = -\frac{\cos^m x \cos nx}{n} - \frac{m}{n} \underbrace{\int \cos^{m-1} x \sin nx \cos x}_{I_{m-1,n}} + \frac{m}{n} \underbrace{\int \cos^{n-1} x \sin(n-1)x\, dx}_{I_{m-1,n-1}}$$

$$I_{m,n} = \frac{-\cos^m x \cos nx}{n} - \frac{m}{n} I_{m,n} + \frac{m}{n} I_{m-1,n-1}$$

$$\left(1+\frac{m}{n}\right) I_{m,n} = \frac{-\cos^m x \cos nx}{n} + \frac{m}{n} I_{m-1,n-1}$$

$$\frac{(m+n)}{\cancel{n}} I_{m,n} = \frac{-\cos^m x \cos nx}{\cancel{n}} + \frac{m}{\cancel{n}} I_{m-1,n-1}$$

$$I_{m,n} = \frac{-\cos^m x \cos nx}{(m+n)} + \frac{m}{(m+n)} I_{m-1,n-1}$$

Q: Evaluate $\int_0^{\pi/2} \cos^5 x \sin 3x \, dx$

$m=5,\ n=3$

<u>Solu^n</u> $\Rightarrow$ $I_{5,3} = -\dfrac{\cos^5 x \cos 3x}{8}\Bigg|_0^{\pi/2} + \dfrac{5}{8}\int_0^{\pi/2} \cos^4 x \sin 2x \, dx$

let $\cos x = t$

$-\sin x \, dx = dt$

$$= \frac{1}{8} + \frac{5}{8} I_{4,2} \quad — (1)$$

$= \frac{1}{8} + \frac{5}{8} \times \frac{1}{3} = \frac{1}{3}$ Ans

$-\int_1^0 t^3 \, dt = \frac{t^4}{4}$

$$I_{4,2} = \frac{1}{4+2} + \frac{4}{6}\int_0^{\pi/2} \cos^3 x \sin x \, dx$$

$\frac{3+1}{24} = \frac{4}{24}$

$$= \frac{1}{6} + \frac{\cancel{2}}{3}\left[\frac{1}{{}_2\cancel{4}}\right] = \frac{1}{3} \quad — (2)$$

Q: $\int_0^{\pi/2} \cos^{n-2} x \sin \underline{n}x \, dx$ (H.W.)

Prove $\int_0^{\pi/2} \cos^{n-2} x \cos nx \, dx = \frac{\pi}{2^{n+1}}$

Q: $I_n = \int_0^{\infty} e^{-x} x^n \, dx$, n +ve integ. Obtain a linear relation between I_n & I_{n-1}. Hence, evaluate I_n.