$\bar{z}_1 z_2 + z_1 \bar{z}_2$ is real $\Rightarrow (z_1\bar{z}_2 + \bar{z}_1 z_2)^2 \geq 0$ — (2)

$(z_1\bar{z}_2 - \bar{z}_1 z_2)$ is imag. $\Rightarrow (z_1\bar{z}_2 - \bar{z}_1 z_2)^2 \leq 0$

$$(z_1\bar{z}_2 + \bar{z}_1 z_2)^2 - (z_1\bar{z}_2 - \bar{z}_1 z_2)^2 = 4 z_1\bar{z}_1 z_2\bar{z}_2 = 4|z_1|^2|z_2|^2$$ — (3)

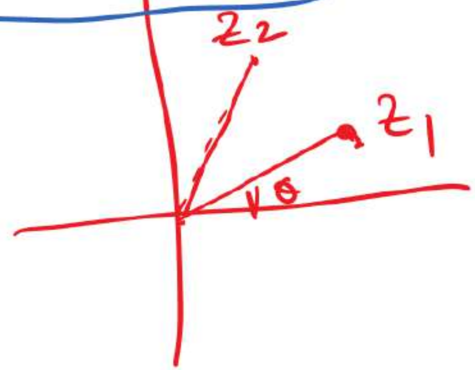
From (2) & (3)

$$(z_1\bar{z}_2 + \bar{z}_1 z_2)^2 \leq 4 |z_1|^2 |z_2|^2$$ — (4)

From (1) & (4) $|z_1 + z_2|^2 \leq |z_1|^2 + |z_2|^2 + 2|z_1||z_2|$

$$\because \boxed{\cos\theta + i\sin\theta = e^{i\theta}}$$

$$= \left[|z_1| + |z_2|\right]^2$$



$$\Rightarrow |z_1 + z_2| \le |z_1| + |z_2|$$

From the preceding result, we can deduce that

$$|z_1 + z_2| \ge \big||z_1| - |z_2|\big|$$

$a^m . a^n$
$= a^{m+n}$

Argumenents: $z_1 = r_1(\cos\theta_1 + i\sin\theta_2)$; $z_2 = r_2(\cos\theta_2 + i\sin\theta_2)$

$z_1 = r_1 e^{i\theta_1}$

$z_2 = r_2 e^{i\theta_2}$

$z_1 \cdot z_2 = r_1 r_2 \cdot e^{i(\theta_1+\theta_2)}$

1) $z_1 z_2 = r_1 r_2 (\cos(\theta_1+\theta_2) + i\sin(\theta_1+\theta_2))$

2) $z_1/z_2 = r_1/r_2 [\cos(\theta_1-\theta_2) - i\sin(\theta_1-\theta_2))$

* $z^n = 1$

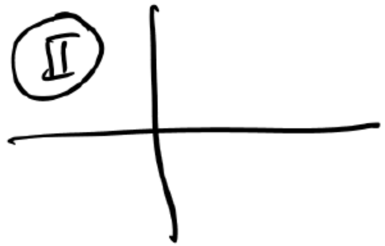
$z^3 = 1$

$1, \omega, \omega^2$

$z = (1)^{1/n}$

$z_0 = (1 \cdot e^{0i}) = \cos 0 + i \sin 0 = 1$

$z = e^{\frac{2k\pi i}{n}}$; $k = 0, \dots, n-1$



$\boxed{z^3 = 1}$

$z = e^{\frac{2k\pi i}{3}}$, $k = \underbrace{0, 1, 2}$

$z_0 = e^{0i} = 1$

$z_1 = e^{\frac{2\pi}{3} i} = \cos \frac{2\pi}{3} + i \sin \frac{2\pi}{3} = -\frac{1}{2} + \frac{\sqrt{3}}{2} i$

$$z_2 = e^{i\frac{4\pi}{3}} = -\frac{1}{2} - \frac{\sqrt{3}}{2} i$$

$$z_0 + z_2 + z_3 = 1 + \frac{1}{2} + \cancel{\frac{\sqrt{3}}{2}} i - \frac{1}{2} - \cancel{\frac{\sqrt{3}}{2}} i = 0$$

$$\boxed{1 + \omega + \omega_2 = 0} \qquad \omega^3 = ?$$

Q: If the sum and product of two complex nos. are both real then the two nos. must either be real or conjugate.

Soln: $z_1 = x_1 + iy_1$ & $z_2 = x_2 + iy_2$

$$z_1 + z_2 = (x_1 + x_2) + i\,(y_1 + y_2)$$

i.e. $y_1 + y_2 = 0$ (given)

$$\Rightarrow \boxed{y_1 = -y_2 \quad - (1)}$$

$$z_1 \cdot z_2 = (x_1 + iy_1) \cdot (x_2 + iy_2) = x_1 x_2 - y_1 y_2 + i\,(x_1 y_2 + x_2 y_1)$$

i.e. $x_1 y_2 + x_2 y_1 = 0$ or $\dfrac{x_1}{x_2} = \dfrac{-y_1}{y_2} = 1$

$$\Rightarrow \boxed{x_1 = x_2}$$

$$z_2 = x_1 - iy_1 = \overline{z_1}$$