* Deduce Planck's law of radiation or Planck's formula for radiation. Also derive Wien's law and Rayleigh-Jeans law.

Planck introduced the theory of quanta to radiation. According to this, radiant energy is in the form of very small particles called photones. The energy of these particles are integral multiples of $h\nu$. i.e; it may be $0 \times h\nu$, $1 \times h\nu$, $2 \times h\nu$, ..., $n \times h\nu$.
Here $h = 6.64 \times 10^{-27}$ erg.sec,
ν = frequency of radiation.

The aim of Planck's law is to find energy density of radiation in frequency range ν and $\nu + d\nu$ or wavelength range λ and $\lambda + d\lambda$.

For deriving the law, we consider the black body chamber filled with particles of radiation and perfect gas. There is energy exchange between the two types of particles. Due to presence of ideal gas molecules Maxwell-Boltzmann distribution law is valid. This law has the form

$$A \cdot e^{-\epsilon/KT} \cdot \cancel{K}$$

where $\epsilon = nh\nu$,
K = Boltzmann constant, T = temperature.

According to this, the number of particles per unit volume having quantized energy value are expressed by

$$A \cdot e^{-0 \cdot h\nu/KT} \cdot d\epsilon,\ A \cdot e^{-1 \cdot h\nu/KT} \cdot d\epsilon, \ldots\ldots$$

$$\ldots\ldots\ldots,\ A \cdot e^{-nh\nu/KT} \cdot d\epsilon, \ldots$$

The ratio of energies is given by

$$1 : e^{-h\nu/KT} : e^{-2h\nu/KT} : \ldots\ldots\ldots$$

If M = number of particles or resonaters or vibrators have zero energy, then the particles having energies in quantized value is given by

$$M \cdot e^{-h\nu/KT},\ M \cdot e^{-2h\nu/KT}, \ldots\ldots\ldots$$

$$\ldots\ldots,\ M \cdot e^{-\alpha h\nu/KT}$$

Total number of resonators is expressed as the sum

$$N = M \cdot e^{-0 \cdot h\nu/KT} + M \cdot e^{-1 \cdot h\nu/KT} + \ldots\ldots$$

$$\ldots\ldots + M \cdot e^{-\infty h\nu/KT} + \ldots$$

$$= M\left(1 + e^{-h\nu/KT} + e^{-2h\nu/KT} + \ldots\ldots\right)$$

[This is inf. G.P.]

$$\text{or, } N = M \cdot \frac{1}{1 - e^{-h\nu/kT}} \quad \ldots\ldots\ldots\ldots \text{(i)}$$

Due to quantized energy values the total energy of the particles are is given by

$$U = M \cdot e^{-0h\nu/kT} \times 0h\nu + M \cdot e^{-h\nu/kT} \times 1h\nu$$

$$+ M \cdot e^{-2h\nu/kT} \times 2h\nu + \ldots\ldots\ldots$$

$$= M \cdot e^{-h\nu/kT} \cdot h\nu \left[1 + 2e^{-h\nu/kT} + 3e^{-2h\nu/kT} + \ldots \right]$$

$$\text{or, } U = M \cdot e^{-h\nu/kT} \cdot h\nu \left[1 - e^{-h\nu/kT}\right]^{-2}$$

$$\text{or, } U = \frac{M \cdot e^{-h\nu/kT} \cdot h\nu}{\left(1 - e^{-h\nu/kT}\right)^2} \quad \ldots\ldots\ldots\ldots \text{(ii)}$$

Therefore, average energy per vibrational mode or per particle is given by

$$\frac{U}{N} = \epsilon_{av} = \frac{M \cdot e^{-h\nu/kT} \cdot h\nu}{\left(1 - e^{-h\nu/kT}\right)^2} \bigg/ \frac{M}{\left(1 - e^{-h\nu/kT}\right)}$$

$$\text{or, } \boxed{\frac{U}{N} = \epsilon_{av} = \frac{h\nu}{e^{h\nu/kT} - 1}} \quad \ldots\ldots \text{(iii)}$$

According to Rayleigh-Jeans law, the number of vibrational modes per unit volume is given by $\frac{8\pi\nu^2 \cdot d\nu}{c^3}$ in terms of frequency and $\frac{8\pi}{\lambda^4} \cdot d\lambda$ in terms of wavelength.

Therefore, using this with Planck's average energy per particle, the energy density distribution law becomes

$$u_\nu \, d\nu = \frac{8\pi \nu^2 \cdot d\nu}{c^3} \times \frac{h\nu}{e^{h\nu/KT}-1} \quad \ldots\ldots (iv)$$

This is Planck's radiation formula in terms of frequency. In terms of wavelength, the formula becomes

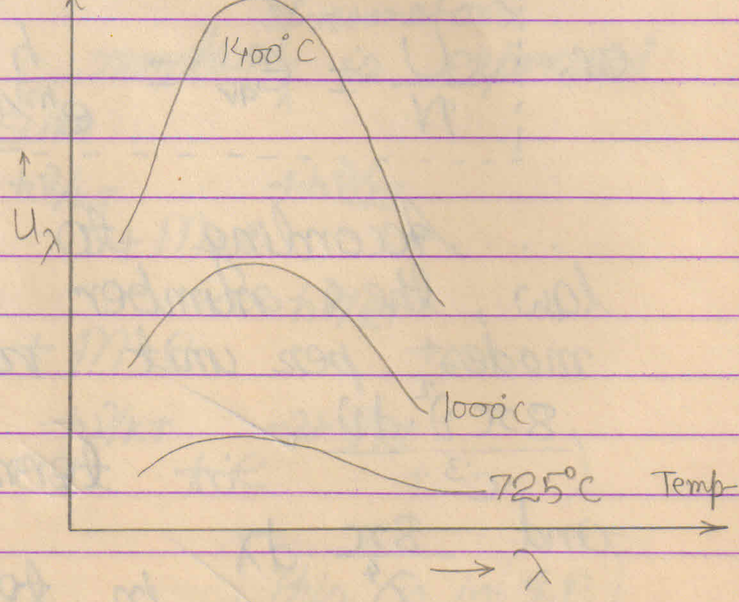
$$\boxed{u_\lambda \, d\lambda = \frac{8\pi hc}{\lambda^5} \cdot d\lambda \times \frac{1}{e^{hc/\lambda KT}-1}} \quad \ldots (v)$$

where we have used transformation relation:

$$\nu = c/\lambda \quad \therefore \; h\nu = hc/\lambda$$

$$\text{and } d\nu = c \cdot \frac{d\lambda}{\lambda^2}$$

If, we compare this law with experimental results of Lummer and Pringshiem, then the whole range of spectrum is explained.



Derivation of Wien's law from Planck's law.

For deriving Wien's law from Planck's law, we consider the form of Planck's law in terms ~~form~~ of wavelength by writing

$$\frac{hc}{\lambda KT} = x$$

such that, $hc = \lambda K T x$

The Planck's law becomes

$$u_\lambda \cdot d\lambda = \frac{8\pi hc}{\lambda^5} \cdot \frac{1}{e^x - 1} \cdot d\lambda \quad \ldots\ldots \text{(vii)}$$

Incase both λ and T are small, the product λT will be smaller. Therefore, e^x is very large quantity. Therefore, unity may be neglected in comparision to e^x and the law becomes

$$u_\lambda \cdot d\lambda = 8\pi hc \lambda^{-5} \cdot e^{-hc/\lambda KT} \cdot d\lambda \quad \ldots \text{(viii)}$$

In this if we ~~have~~ write

$$\left.\begin{aligned} 8\pi hc &= C_1 \\ \text{and} \quad hc/\lambda &= C_2 \end{aligned}\right\} \quad \ldots\ldots \text{(ix)}$$

Then $u_\lambda d\lambda = C_1 \lambda^{-5} e^{-C_2/\lambda KT} \cdot d\lambda$

This is Wien's law

~~These are Rayleigh-Jeans constant.~~ and C_1 and C_2 are Wien's Constants

~~Equation (viii) is proper form of Wien's law~~ Thus, for low temperature small wavelength, Planck's law reduces to Wien's law of radiation.

R.J. Law: ~~On the other hand~~ when both λ and T are large, ~~then~~

λT will be still heigher. $x \rightarrow$ Small Value.

In this case

$$e^x - 1 = \left[1 + x + \frac{x^2}{2!} + \ldots\ldots -1\right]$$

$$\simeq x \qquad \left[\text{Since } x \text{ is small, } x^2 \text{ and etc} \rightarrow 0\right]$$

The Planck's law now becomes

$$u_\lambda \cdot d\lambda = \frac{8\pi \cdot \lambda k T x}{\lambda^5} \cdot \frac{1}{e^x - 1} \cdot d\lambda$$

$$= \frac{8\pi}{\lambda^4} \cdot d\lambda \cdot kT \ldots\ldots (x)$$

This is Rayleigh - Jeans law or classical law of radiation.

$e^{h\nu/kT}$