Symmetric and Antisymmetric Tensors

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Introduction

Tensors are mathematical quantities used to describe physical properties such as stress, strain, angular momentum, and electromagnetic fields. Scalars are tensors of rank zero, vectors are tensors of rank one, and matrices represent tensors of rank two.

Symmetric Tensor

A second-rank tensor Tij is called symmetric if Tij = Tji.

In three dimensions, a symmetric tensor has 6 independent components.

Examples:

Moment of inertia tensor, stress tensor, strain tensor, metric tensor.

Antisymmetric Tensor

A tensor Aij is antisymmetric if Aij = –Aji. All diagonal elements are zero.

In three dimensions, an antisymmetric tensor has 3 independent components.

Examples:

Angular momentum tensor, electromagnetic field tensor, rotation tensor.

Decomposition of Tensor

Any tensor Tij can be written as the sum of symmetric and antisymmetric parts:

$T_{ij} = \frac{1}{2}(T_{ij} + T_{ji}) + \frac{1}{2}(T_{ij} - T_{ji})$

Conclusion

Symmetric tensors describe equilibrium and deformation, while antisymmetric tensors describe

rotation and circulation. Both are essential in mathematical physics.