* What are the postulates of S.T.R. (special theory of relativity)? Derive Lorentz - Einstein transformation relations.

The postulates of S.T.R. are:-

i) The velocity of light in vacuum is constant independent of direction of propagation and relative velocity of the source and observer. This is constancy of velocity of light and,

ii) The principle of equivalence : The mechanical laws are equivalent in all ~~or~~ inertial frames of reference.

Based on these two postulates we consider static reference system S with co-ordinates (O, x, y, z) and moving reference system S′ with co-ordinates (O', x', y', z') which coinside at

$$t = t' = 0$$

Motion of S′ is v along common x-axis. This is indicated in figure 1.

In this reference the Lorentz - Einstein transformations are expressed as,

$$x' = \frac{x - vt}{\sqrt{1 - \frac{v^2}{c^2}}} \quad \ldots\ldots\ldots \text{(i)}$$

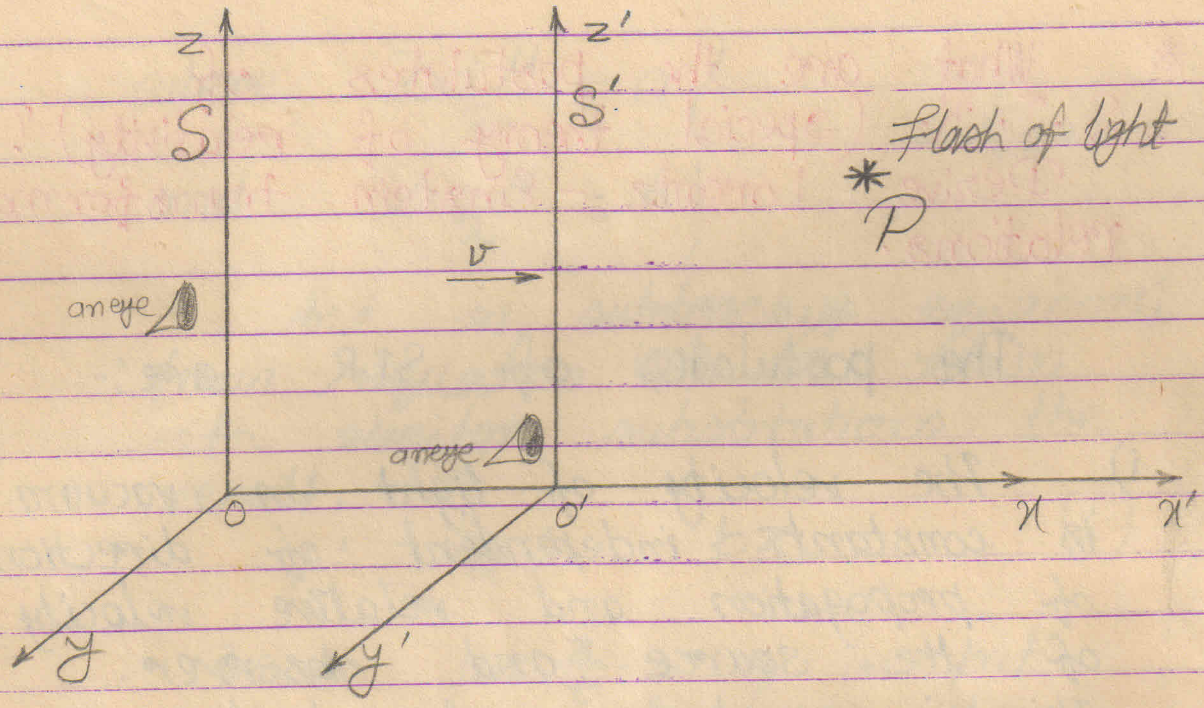

Figure 1 At $t = t' = 0$
O' is at O

$$\left.\begin{aligned} y' &= y \\ z' &= z \end{aligned}\right\} \quad \ldots\ldots\ldots \text{(ii)}$$

$$t' = \frac{t - \frac{vx}{c^2}}{\sqrt{1 - \frac{v^2}{c^2}}} \quad \ldots\ldots\ldots \text{(iii)}$$

These are known as direct Lorentz - Einstein transformation relations.

The inverse transformation relations are obtained by changing v to $-v$ and primed quantities with unprimed.

$$x = \frac{x' + vt'}{\sqrt{1 - \frac{v^2}{c^2}}} \quad \ldots\ldots\ldots \text{(iv)}$$

$$\left.\begin{aligned} y &= y' \\ z &= z' \end{aligned}\right\} \quad \ldots\ldots\ldots \text{(v)}$$

$$t = \frac{t' + \frac{vx'}{c^2}}{\sqrt{1 - \frac{v^2}{c^2}}} \quad \ldots\ldots\ldots \text{(vi)}$$

With flash of light at point P, the observer in static frame S receives the signal after time t which is expressed by,

$$x = ct$$

or, $x - ct = 0$ .... (1a)

The same event for observer in S' is described by,

$$x' = ct'$$

or, $x' - ct' = 0$ .... (1b)

where c is velocity of light.

According to principle of equivalence we have,

$$x' - ct' = \lambda(x - ct) \quad \text{.........} (1)$$

This is the relation for positive direction of x-axis.

Considering negative direction of x-axis we obtain the equivalence relation with changed sign and the result is,

$$x' + ct' = \mu(x + ct) \quad \text{.........} (2)$$

where λ and μ are separate constants.

In solving these for x' and t' we first add and obtain,

$$2x' = (\lambda + \mu)x - (\lambda - \mu)ct$$

$$\text{or} \quad x' = \frac{(\lambda + \mu)}{2}x - \frac{(\lambda - \mu)}{2}ct$$

This is written as,

$$x' = ax - bct \quad \text{.........} (3)$$

where $\frac{\lambda+\mu}{2} = a$

and $\frac{\lambda-\mu}{2} = b$ $\Bigg\}$ ....(3a)

And on subtracting equation (1) from equation (2) we obtain with similar substitutions the result,

$$t' = at - \frac{bx}{c} \quad \text{.......(4)}$$

Since $v = \frac{x}{t}$, if we put $x'=0$ in equation (3) then we obtain,

$$x = \frac{bct}{a} \quad \text{..........(4a)}$$

And the relative velocity of S' with reference to S is given by,

$$V = \frac{x}{t} \quad \text{..........(4b)}$$

Eleminating x from (4a) and (4b) we obtain,

$$v = \frac{bc}{a} \quad \text{..........(5)}$$

and. $b = \frac{va}{c}$ ....(5a)

Since origins O and O' coinside at $t = t' = 0$; substituting $t=0$ means measurement from S about the distance in S'. Similarly, putting $t'=0$ in equation (4) indicates measurment of distances in S from S'.

First condition $t=0$ used in equation (3) gives,

$$x' = ax$$

or, $\Delta x' = a \cdot \Delta x$

If $\Delta x' = 1$ then, we obtain,

$$\Delta x = \frac{1}{a} \quad \ldots\ldots\ldots\ldots (6)$$

This means a unit distance in moving reference frame appears as $\frac{1}{a}$ in static reference S.

Using second condition, $t'=0$ in equation (6) we obtain,

$$act = bx$$

Substituting in this the value of b from equation (5a) we obtain.

$$t = \frac{vx}{c^2} \quad \ldots\ldots\ldots\ldots (6a)$$

Now substituting this value of t from equation (6a) in equation (3) we obtain,

$$x' = ax - \frac{va}{c} \cdot c \cdot \frac{vx}{c^2}$$

or, $x' = a\left(1 - \frac{v^2}{c^2}\right)x$

This gives,

$$\Delta x' = a\left(1 - \frac{v^2}{c^2}\right)\Delta x$$

In this, if we put the static distance $\Delta x = 1$ then we obtain,

$$\Delta x' = a\left(1 - \frac{v^2}{c^2}\right) \quad \ldots\ldots\ldots\ldots (7)$$

Now the same unit distance is read by equation (6) and equation (7). Equating the two we obtain,

$$\frac{1}{a} = a\left(1 - \frac{v^2}{c^2}\right)$$

This gives,

$$a = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \quad \ldots\ldots\ldots (8)$$

Now Substituting the value of b from equation (5a) and that of a from equation (8) in equation (3) we obtain,

$$x' = \frac{x}{\sqrt{1 - \frac{v^2}{c^2}}} - \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \cdot \frac{v}{c} \cdot ct$$

This gives,

$$x' = \frac{x - vt}{\sqrt{1 - \frac{v^2}{c^2}}} \quad \ldots\ldots\ldots\ldots (i)$$

This is Lorentz – Eienstein transformation relation for distance.

Now substituting the value of a from equation (8) and b from (5a) in equation (4) we obtain,

$$t' = \frac{t}{\sqrt{1-\frac{v^2}{c^2}}} - \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} \cdot \frac{v}{c} \cdot \frac{x}{c}$$

$$\text{or, } t' = \frac{t - \frac{vx}{c^2}}{\sqrt{1-\frac{v^2}{c^2}}} \quad \ldots\ldots\ldots\ldots (iii)$$

This is Lorentz-Einstein transformation relation for time.

Since motion of S' has been considered in x-direction alone, Y and Z are uneffected and we have,

$$Y' = Y$$
$$\text{and } Z' = Z$$

On reversing the sign of v to $-v$ and interchange in the primes on variable we obtain the inverse Lorentz-Einstein transformation relations expressed by equations (iv, v and vi)

~~Inverse~~ ~~Rewrite~~