The Lorentz-Fitzgerald Contraction Hypothesis

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In the previous note on Michelson-Morley experiment (*Read that note before reading this. We use here same symbols as was used in the previous note for easy access to the material.*), we have discussed that Michelson-Morley couldn't verify the existence of ether hypothesis. They performed the experiment during all seasons of a year as the Earth rotates around the Sun and as well as during many times at day and night with respect to earth's spin rotation around its axis. Thus, they taken a lot of observations with changing Earth's velocity with respect to ether. But they couldn't found fringe shift. However, their theoretical calculation showed that there should be fringe shift. The conclusion of the result could be drawn that light speed should be same in all directions. But this violates the Galilean transformation rule which was the very much accepted and well established theory of the time. People believed that there should be an alternate hypothesis which should explain the result of Michelson-Morley experiment and preserve the ether hypothesis.

In order to explain the Michelson-Morley null result with the accepted ether hypothesis at time, in 1892, Fitzgerald proposed a hypothesis which was further developed by Lorentz. The basic idea of their hypothesis was that when a body moves relative to the stationary ether *it is contracted in the direction of motion by a factor of* $\sqrt{1-\beta^2}$*, where* β *is the ration of object velocity to the light velocity, i.e.,* $\beta = u/c$.

Thus, according to the hypothesis, If l^r represents the rest length with respect to ether and l is the length in motion with respect to ether, then relation between l and l^r is given by

$$l = l^r\sqrt{1-\beta^2}, \tag{1}$$

where $\beta = u/c$.

In the previous note of Michelson-Morley experiment, we have seen that the the time

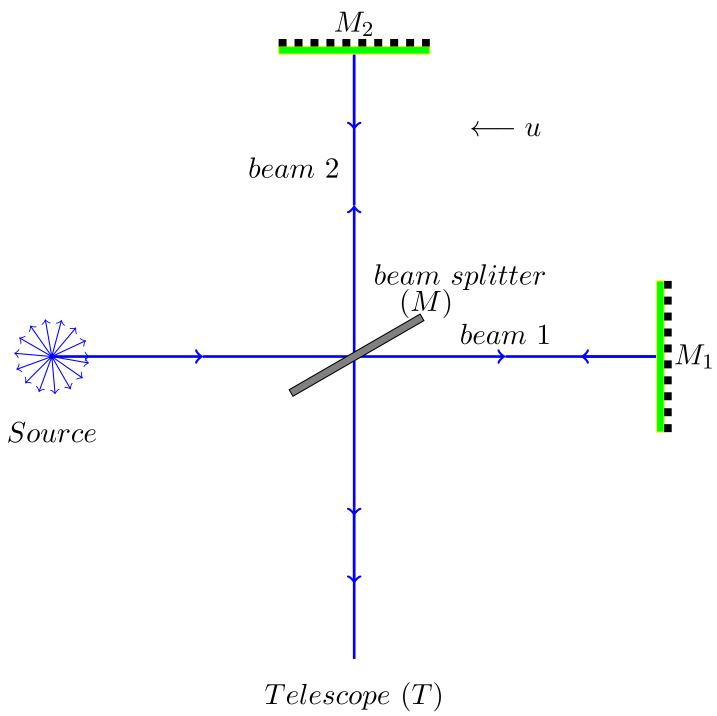

FIG. 1: Michelson Interferometer. u denotes the velocity of ether with respect to the Interferometer. M denotes the partially silvered mirror which is used to split the beam coming from source into two beams. The beams are reflected back from mirrors M_1 and M_2. The reflected beams transmitted to the telescope where one obtains the fringe patterns due to interference of beams.

difference between the two beams before 90^0 rotation of the apparatus (see Figs. (1 and (2)) is given by

$$\Delta t = \frac{2}{c}\left(\frac{l_2}{\sqrt{1-u^2/c^2}} - \frac{l_1}{1-u^2/c^2}\right). \tag{2}$$

Rotating the apparatus by angle 90^0,

$$\Delta\tau = \frac{2}{c}\left(\frac{l_2}{1-u^2/c^2} - \frac{l_1}{\sqrt{1-u^2/c^2}}\right). \tag{3}$$

Now using Eq. (1) in the above two equations, we obtain the corresponding time differences denoted by Δt_{LF} and $\Delta\tau_{LF}$ corresponding to the Lorentz contraction as

$$\Delta t_{LF} = \frac{2}{c}\frac{1}{\sqrt{1-\beta^2}}(l_1^r - l_2^r), \tag{4}$$

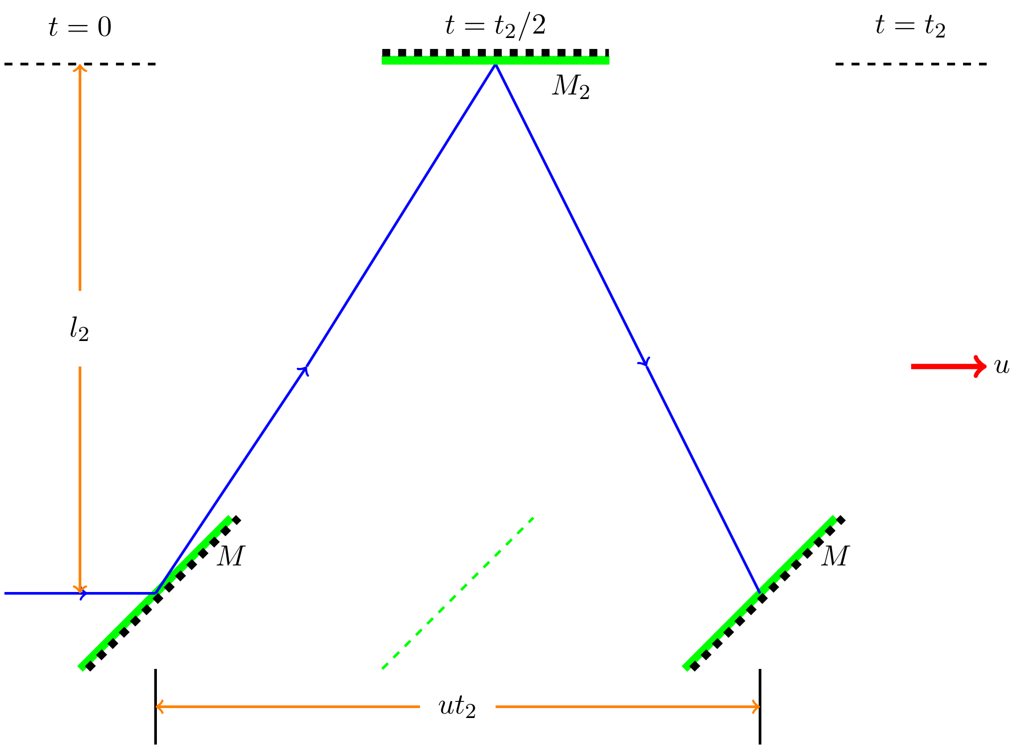

FIG. 2: Figure demonstrates the cross stream path of beam 2. Here mirrors move with velocity u with respect to ether, also u is the velocity of Interferometer with respect to ether.

and

$$\Delta\tau_{LF} = \frac{2}{c}\frac{1}{\sqrt{1-\beta^2}}(l_1^r - l_2^r). \tag{5}$$

From Eqs. (4) and (5), we see that

$$\Delta\tau_{LF} - \Delta t_{LF} = 0, \tag{6}$$

which indicates that there is no shift in fringe pattern due to rotation. But story is not finished yet. Suppose velocity of the apparatus/ Interferometer changes from u to u' with respect to ether (Here we consider $l_1 \neq l_2$), then we get a fringe shift

$$\Delta S = \nu(\Delta\tau - \Delta t) = \frac{(l_1^r - l_2^r)}{\lambda}\left[\frac{u^2}{c^2} - \frac{u'^2}{c^2}\right]. \tag{7}$$

For the case of equal arm length, ΔS is zero but we have considered $l_1 \neq l_2$. The result (7) says that ΔS is nonzero. The factor $(u^2 - u'^2)/c^2$ depends on the Earth's spin and orbital rotation. But people at the time didn't observe any effect experimentally due to the factor.

[1] R. Resnick, *Introduction to Special Relativity*, Wiley-VCH, (1968) .