Newtonian Relativity

By

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Historical background

- Newtonian mechanics 1600's
- Unification of Electricity & Magnetism: Maxwell's equations 1865
- A A Michelson and E W Morley experiment 1887
- Special theory of relativity 1905
- General theory of relativity 1916

Motivation: a physicist point of view

- What will be the momentum and energy conservation laws for two inertial observers, say one observer on the ground and another in a moving train (acceleration=0), observing a collision of two objects? Will it be same or different for both the observers?
- Can we obtain some transformation rule such that by studying the phenomena of one event in a particular frame we can describe the same event in another frame?

Motivation: a physicist point of view

- To describe an event what are the mathematical tools we need to develop?
- How one can establish relations between quantities of one frame to the other frame?

Outline

1. Frame of reference
   - Inertial frames
   - Non-inertial frames
2. Galilean transformation
   - Time and space interval under Galilean transformation
     - Time interval
     - Length
   - Velocity addition theorem
   - Acceleration transformation rule
3. Some questions

Frame of reference

A frame of reference is a set of coordinates which are used to specify an *event* such as collision of two objects, turning on/off a flashlight, etc.

Two categories of reference frames:

- Inertial
- Non-inertial

Inertial frames

- An inertial system can be treated as a frame in which *Newton's first law which is also known as the law of inertial*, holds. These frames are called as *inertial reference frames*.
- An unaccelerated system, i.e., a system *moving with constant velocity* with respect to an inertial frame is also called as inertial reference frame.

- Some more examples:
  - ⇒ A set of axes fixed on earth is treated as the inertial coordinate system.

    Note: While treating earth as inertial reference one ignores the small acceleration effect due to rotation (around its axis) and orbital motion of the earth around the sun.
  - ⇒ Any set of axes moving with constant velocity with respect to earth is treated as inertial frame. For example, a train/car/bus/van/tractor/motorcycle/boat/ship/helicopter/airplane moving with constant velocity with respect to ground is inertial reference frame.

Non-inertial frames

A frame of reference *accelerating* with respect to an inertial system is called as *non-inertial reference frame.*

Examples: An accelerating car, two sphere tied with a string and rotating about each other, etc.

Galilean transformation

An event is specified by point P in the figure below.

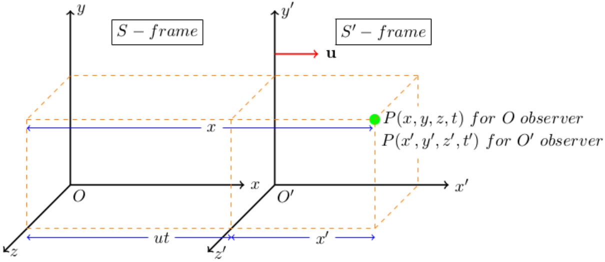

Figure 1: Inertial frames. S' frame moving with constant velocity $\mathbf{u}$ relative to S frame in the +ive $x-$direction, where $\mathbf{u} = (u, 0, 0)$. Both the frames have common x-x'-axis and other axes y-y' and z-z' of the two frames are parallel. Initially at $t = t' = 0$, both the frames were coinciding at the origin O. After some time t, the $S' - frame$ moved to a distance ut in the +ive $x-$direction.

Space-time transformation relations between the two frames (Fig. (1)):

$$x' = x - ut,$$
$$y' = y,$$
$$z' = z,$$
$$t' = t\ .$$

The above transformation relation is called as Galilean transformation.

We can write the above equations in compact form as

$$\mathbf{x}' = \mathbf{x} - \mathbf{u}t; \quad t' = t.$$

Time interval

Time interval between two events A and B are invariant under Galilean transformation, i.e., interval is same for both the observers. From Galilean transformation rule

$$t'_A - t'_B = t_A - t_B.$$

Length

We consider a rod which is at rest in S frame and whose ends points are P and Q. Using Galilean transformation relation

$$x'_Q - x'_P = x_Q - x_P - u(t_Q - t_P).$$

Since the end points of the rod P and Q are *measured at the same time* $\Longrightarrow t_Q = t_P$. Now we obtain

$$x'_Q - x'_P = x_Q - x_P \Longrightarrow \text{length is invariant.}$$

Velocity addition theorem

Differentiating the space transformation rule of Galilean transformation with respect to t we obtain,

$$\begin{aligned} v'_x &= v_x - u, \\ v'_y &= v_y, \\ v'_z &= v_z, \end{aligned} \tag{1}$$

where $dx/dt = v_x$, $dy/dt = v_y$, and $dz/dt = v_z$ and similarly we define corresponding velocity components in the primed frame.

Vectorial form:

$$\mathbf{v}' = \mathbf{v} - \mathbf{u}. \tag{2}$$

The above equation relates the object velocity in the two frames.

Acceleration transformation rule

Differentiating the velocity transformation rules, obtained above, with respect to t we obtain,

$$\begin{aligned} \frac{dv'_x}{dt'} &= \frac{dv_x}{dt} \Rightarrow a'_x = a_x, \\ \frac{dv'_y}{dt'} &= \frac{dv_y}{dt} \Rightarrow a'_y = a_y, \\ \frac{dv'_z}{dt'} &= \frac{dv_z}{dt} \Rightarrow a'_z = a_z. \end{aligned} \tag{3}$$

We see that $\mathbf{a}' = \mathbf{a}$.

Conclusion: *Newton's laws of motion, equations of motion of a particle, conservation principles of mechanics such as linear momentum, angular momentum, energy are same in all inertial frames. This implies that* **laws of mechanics are invariant under Galilean transformation**.

Some questions

From Maxwell's electromagnetic field equations one can obtain the wave equation given by

$$\nabla^2 \mathbf{\Phi} = \frac{1}{c^2}\frac{\partial \mathbf{\Phi}}{\partial t}, \tag{4}$$

where $\mathbf{\Phi}$ denotes the fields either $\mathbf{E}$ or $\mathbf{B}$.

Using the Galilean transformation rule in the above equation one can verify that equations are not invariant.

Galilean transformation is violated for Maxwell's equations. This gives rise some interesting questions about Newtonian relativity.

- Is Newtonian relativity invalid?
- Or Newtonian relativity is valid but Maxwell's electromagnetic equations are wrong?
- Or Newtonian relativity is valid only for mechanics?

The above queries can be answered by the Einstein's relativity theory. But suppose if someone is unaware of Einstein's relativity theory what could be his/her answers to above queries?

Next time we will discuss Einstein's relativity theory!

References

- R. Resnick, *Introduction to Special Relativity*, Wiley-VCH, (1968) .