Why do we need Quantum Mechanics?

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1 Inadequacies of Classical Mechanics

Classical mechanics (also well-known as Newtonian Mechanics) is based upon mainly three laws: 1. Law of Inertia (Newton's first law) 2. Law of Force (Newton's second law) 3. Law of Action-Reaction (Newton's third law). Classical mechanics can successfully explain the motions (translational, rotational and combined) of finite sized objects, predict their trajectories and momentum upon dealing with the forces that act on the objects. Therefore, if you consider the motion of the earth, rotating around the sun under the action of central force ('gravity' in this case), the classical mechanics will give you the exact position ($\overrightarrow{r}$,θ) of the earth at any given instant of time (t). However, if you apply the same principle inside an atom, considering the electrostatic force between the positively charged nucleus and the negatively charged electron as central force, you can never determine the exact position of the electron! Obviously, calculations based upon classical considerations will give you definite mathematical value for the position of the electron at a given time. But the result is misleading. Here, the classical theory fails (The reason can be explained using Heisenberg's uncertainty principle and will be discussed later.) Now, think of an oscillating bob, suspended from the ceiling by an attached massless string of length l (a simple pendulum). For such a simple harmonic motion (SHM) the period of oscillation $T = 2\pi\sqrt{\frac{l}{g}}$, where g is gravitational acceleration (g=9.81 m/s^2). Evidently, T depends on the length l of the string and therefore it can take any value based upon the magnitude of l. In fact, if l is continuous variable, T also changes continuously. Hence, the associated angular frequency $\omega(=\frac{2\pi}{T})$ is also a continuous parameter. On the other hand, when the atoms of a linear diatomic molecule vibrate periodically around the center of mass (stretching and compressing along bond-axis), the motion is (upto a certain extent) harmonic oscillation. However, the frequency of oscillation for such type of movement is discrete. Which means, unlike the case of the simple pendulum the frequencies of molecular vibrations are non-continuous i.e. their magnitude may take only specific predetermined values. Again, the classical mechanics fails to explain the observed discreteness. Here, I shall have brief discussions on few significant physical phenomena, without going into their detailed descriptions, illustrations and mathematical derivations, only to shed some light on their quantum mechanical aspects.

1.1 The Blackbody Radiation

A blackbody is a physical object that absorbs all the radiations incident on it, and reflects or transmits none. Practically, an ideal blackbody does not exist. But, there are materials which absorb a major portion of the light incident on them. For example, VANTABLACK (Vertically Alligned NanoTube Arrays BLACK), a nanomaterial that can absorb more than 99.9% of the incident visible rays. For conceptual purpose, let us consider a double-walled hollow spherical shell with Lamp Black coated on its inner surface (see figure 1). Incident radiation enters into the shell through an small aperture and is completely absorbed at the inner wall *via* multiple reflections on the surface. Therefore, this type of cavity may be considered as blackbody good enough for experimental purpose. At finite temperature, thermal equilibrium is attained when the rate of emission of electromagnetic waves from the blackbody becomes equal to the rate of absorbtion. Blackbody radiation emission spectrum contains an wide range of emission wavelengths. In figure 2, typical blackbody radiation spectra are shown for three different temperatures (T=5000, 4000 and 3000 K), where x-axis denotes the wavelengths λ of the emitted electromagnetic waves and y-axis indicates corresponding power density u(λ) or intensity. Notably, the emission intensity increases significantly at higher temperature. At each temperature, initially the radiation curve rises sharply (following the power law) to reach a maximum value followed by an exponential fall-down of intensity at higher λ values. Let us define the

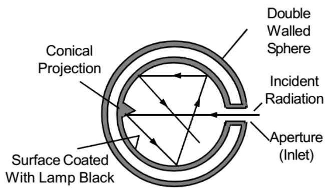

Figure 1: An experimental Blackbody cavity

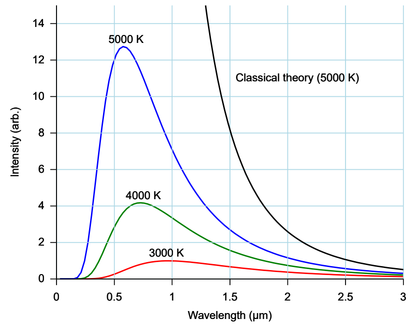

Figure 2: Blackbody radiation curve

wavelength corresponding to maximum emission intensity as λ_{max}. We observe the value of λ_{max} shifts towards lower λ with higher sample temperature T. Wien observed that

$$\lambda_{max} \propto \frac{1}{T}$$

which follows the equation:

$$\lambda_{max} T = Constant$$

, called Wien's displacement law. To explain the blackbody radiation curve theoretically, i.e. to derive a mathematical expression to describe the blackbody radiation intensity as function of wavelength for a fixed temperature, Lord Rayleigh and J. H. Jeans took help of the well-established classical concept. They assumed that the atom/molecules on the inner surface of the cavity behave as classical oscillators at finite temperature where their oscillation creates standing waves within the cavity. The mathematical details of Rayleigh Jean's classical approach will be discussed elsewhere. Here, we directly go for the result of their calculations, which gives energy density u_ν in terms of frequencies,

$$u_\nu = \frac{8\pi\nu^2 kT}{c^2}$$

, where k=Boltzman constant (1.38×10^-23 $m^2kgs^{-2}K^{-1}$), T is the absolute temperature of the blackbody and c is the velocity of light in vacuum. This equation could successfully describe the nature of radiation at lower frequencies (i.e. higher wavelengths). Unfortunately, it fails for higher frequencies (lower wavelengths) which leads to $u_\nu \rightarrow \infty$ as $\nu \rightarrow \infty$. (See the black curve in figure 2) This is called 'Ultravoilet catastrophe'. Wien, on the other hand, derived another classical distribution formula that can explain the blackbody radiation curve successfully within high frequency range (small wavelength), but fails for low frequency side. According to classical theory of radiation, energy changes of the molecular oscillators take place continuously. Max Planck discarded this idea proposing that oscillators can not radiate or absorb energy continuously. Rather, an oscillator of frequency ν can only absorb or radiate energy in unit of magnitude hν, where h is called Planck's constant (h=6.626×10^{-34}J-s). With this idea Planck proposed the molecular oscillators as quantum oscillators. His quantum theory of radiation leads to the following formula:

$$u_\nu = \frac{8\pi h\nu^3}{c^3(e^{\frac{h\nu}{kT}} - 1)}$$

. This formula reduces to Wien's distribution law for high frequency (small wavelength) and it also gives the Rayleigh-Jean's formula at low frequency (high wavelength). It, therefore, can successfully demonstrate the overall behaviour of the blackbody radiation curve throughout full frequency (wavelength) range. One can also derive the Wien's displacement law as well as Stephan Boltzman's law (P=σT^4, power radiated by a blackbody in terms of its temperature) from Planck's quantum hypothesis of blackbody radiation.

1.2 Stability of Atoms

The most simplified form of the atomic structure may be visualized, where a negatively charged electron revolves around the positive nucleus along a circular orbit. If electron's mass is m, velocity is v and orbit radius is r (assuming circular orbit), then the centrifugal force acting on it is $\frac{mv^2}{r}$ and therefore has a centripetal acceleration $\frac{v^2}{r}$. Orbiting electrons are, therefore, continuously accelerating charged particles which, according to classical electromagnetic theory, should emit electromagnetic radiations (Study the Radiation chapter of any Electrodynamics book). Therefore, it is expected that the orbiting electrons will continuously loose energy through radiation and their orbits will shrink gradually. The electron will follow spiral orbits and eventually fall on the nucleus (see figure 3). The atomic structure, in this process, will be destroyed and atoms, therefore, will be unstable. We know that atoms are the building blocks of matter, therefore, matter should be self-destructive and unstable, something we do not observe in reality. Neils Bohr's atomic model provided answers of the questions regarding stability of atoms. He stated that: (1) An

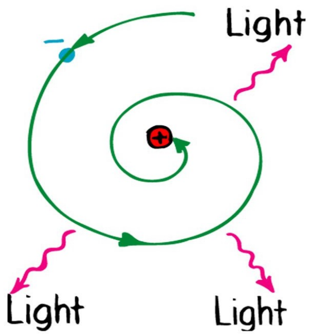

Figure 3: Spiral orbit of electron

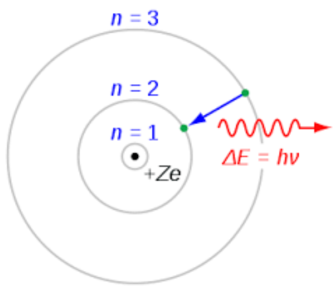

Figure 4: Bohr atomic model

electron can not revolve around the nucleus in all possible orbits as suggested by the classical theory. The electron can revolve around the nucleus only in those allowed orbits for which the angular momentum of the electron is an integral multiple of $\hbar$, where $\hbar = \frac{h}{2\pi}$, h = Planck's constant= 6.626×10^{-34} J.s. Therefore, for an electron with mass m, moving with a velocity v in an orbit of radius r, Angular momentum $L = mvr = n\hbar$, where n=0,1,2,3...called Principal quantum number. (2) If the electron jumps from an initial orbit of energy E_i to a final orbit of energy E_f (E_i>E_f), a photon of frequency

$$\nu = \frac{E_i - E_f}{h} = \frac{\Delta E}{h}$$

is emitted. (See figure 4). Bohr's atomic model supports the stability of atomic structure, moreover, it explains the emission spectrum of hydrogen atom successfully.

1.3 Photoelectric Effect

In 1887 Heinrich Hertz found that some materials absorb electromagnetic waves (light) and emit electrons. These electrons are called 'photoelectrons'. While these photoelectrons passes through an electrical circuit they produce electrical current as may be measured using a micro-ammeter (μA). In figure 5, a typical circuit diagram demonstrates how photoelectric effect is visualized. Light coming from source 'S' is being absorbed at photosensitive plate/cathode 'C'. Photoelectrons emits out of the plate and are being collected on anode 'A'. Electron flow through a closed loop while the photocurrent is measured by μ-Ammeter. This photocurrent (I) can be measured as a function of voltage difference 'V' between plate and anode. Such a typical I-V characteristic is shown schematically in figure 6 for a fixed frequency and intensity of the incident light. At high positive voltages, a saturation plate current is observed, when all the emitted electrons reach the anode. Even at zero voltage of the anode there is a finite measured current. This means that the emitted electrons have their intrinsic kinetic energy $\frac{1}{2}mv^2$. While the anode voltage is biased negatively with respect to the plate, the plate current decreases gradually and eventually becomes zero. The corresponding negative voltage (-V_S) of the anode is called the stopping potential. $|-V_S|$ gives idea about the maximum kinetic energy of the emitted photoelectron $\frac{1}{2}mv^2_{max}$. It was observed later by Lenard that the intensity of the incident light has no effect on the magnitude of stopping potential. Rather, it depends on the frequency of the incident light. In figure 7, the frequency dependance of the stopping potential is shown. In general, the stopping potential increase linearly with the light frequency ν. However, there is a certain frequency ν_{th}, called the 'threshold frequency', below which the stopping potential becomes zero. Interestingly, no photoelectric effect is observed at or below ν_{th}, independent of the intensity of incident light. This phenomenon is quite surprising from the classical

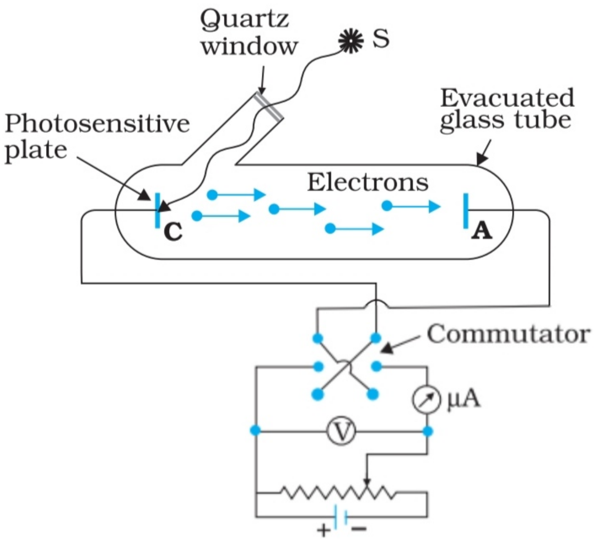

Figure 5: Photoelectric effect circuit

point of view. Because, the well-established classical electrodynamics (based upon Maxwell's equations) has given the idea that electromagnetic wave is composed of sinusoidal electric filed $\overrightarrow{E}$ and magnetic field $\overrightarrow{B}$ where both are mutually perpendicular and also perpendicular to the direction of propagation $\overrightarrow{k}$. The energy density u associated to the electromagnetic wave is :

$$u = \frac{1}{2}\epsilon_0 E^2 + \frac{1}{2}\frac{B^2}{\mu_0}$$

. In other words, the energy of an electromagnetic wave depends on its intensity (I), at least classical mechanics tell us that. So, why the stopping potential V_S (i.e. the maximum kinetic energy of the photoelectron $\frac{1}{2}mv_{max}^2$) depends on frequency ν of the incident light, and not on its intensity? Also, the classical wave model of light does not explain why no photoelectrons are ejected below the threshold frequency ν_{th}, as it predicts that photoelectron must emit if the intensity of the incident light is sufficient enough. In contradiction to this classical theory, it was experimentally observed that even a faint beam (low intensity) of light with frequency higher than ν_{th} can eject photelectrons, while a high intensity beam with frequency lower than ν_{th} can not. This failure of the classical mechanics was resolved by the quantum theory of light invented by Albert Einstein in 1905. Einstein proposed that light is composed of 'energy packets' or 'quanta'. The quanta of light is called 'Photon'. A light of frequency ν is made up of such energy packets or photons where each photon has energy $h\nu$. In brief, quantum theory provides the discrete or particle nature of light. Einstein correlated the photon energy to the maximum kinetic energy of the phtoelectrons by the following relation:

$$h\nu = \frac{1}{2}mv_{max}^2 + \phi$$

, where ϕ is the work function of the plate/cathode material. The above equation may also be written as,

$$h\nu - \phi = \frac{1}{2}mv_{max}^2$$

. If $\phi = h\nu_{th}$, then

$$h(\nu - \nu_{th}) = \frac{1}{2}mv_{max}^2$$

. This equation clearly shows that, for photoemission to occur (i.e. $\frac{1}{2}mv_{max}^2$>0) the condition is $h\nu > h\nu_{th}$, i.e. the incident photon frequency has to be greater than the threshold frequency. This equation also explains, how the stopping potential V_S (or, $\frac{1}{2}mv_{max}^2$) depends linearly on the incident photon frequency.

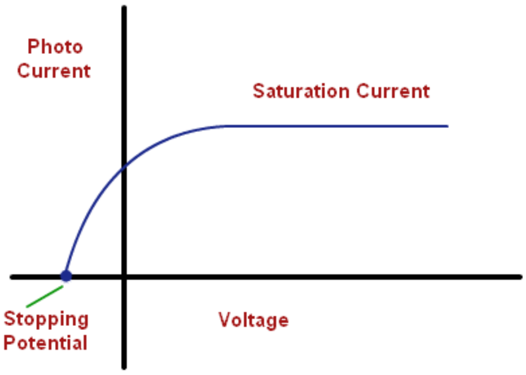

Figure 6: Finding the Stopping Potential

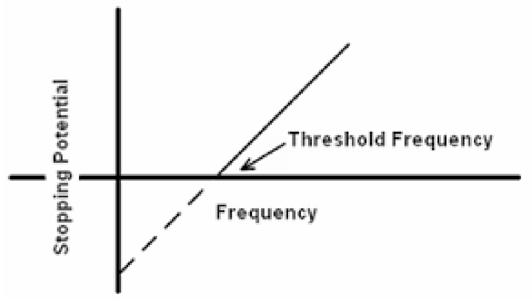

Figure 7: Stopping potential depends on the incident light frequency.

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[1] Figures are collected from Wikipedia and other online resources.